**Excitations in Magnetic Field**

Now we’ll look at various particle in EB field scenarios. Looks like we’ll just neglect the periodic potential and treat free electrons (or could say electrons in conduction band with renormalized mass perhaps). I’m not sure whether or not Bloch’s theorem, or a suitably modified version, still holds if we have magnetic fields?

**Particle in B Field (2D)**

Most everything in here was already covered in the QM folder. But for the sake of comprehensiveness, I review and add a few more details. So consider a spinfull particle in a magnetic field, Bz, in a 2D surface in the xy plane (if want 3D then see QM notes, but generalization is pretty obvious). Note this field would be the bulk-interstitial field, the one felt by the electron under consideration, and so would include both external fields and fields generated by the other particles. We can use either the Landau gauge **A** = (0,Bx,0), or the Symmetric gauge **A** = -(1/2)(**r**×**B**) to describe a magnetic field **B** = B**k**. In the former case we have:



[e can be positive or negative] Might as well quickly solve….Apropos the spatial part, try a wavefunction of the form,



and we end up with the Schrödinger equation,



and so,

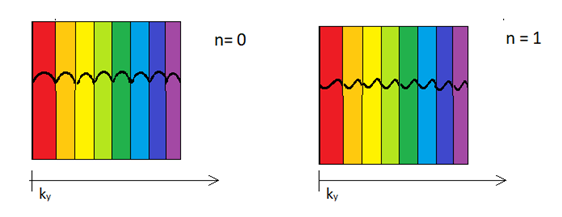


which is the harmonic oscillator equation. And so copying work from the harmonic oscillator file in QM folder, the eigenfunctions/eigenvalues are,



where μB, the Bohr magnetron, is μB = |e|/2m. But should note that the m which appears in ωc (also called ωB) is the effective mass of the electron, derived from the band spectrum, or elsewhere, and is often greater (Cu’s is 30% greater I think) than the rest mass, which appears in ωs. So often ωc < ωs, by a not-insubstantial margin. So I’m going to be careful not to equate the two. And since these masses can be different, we will have a greater variety of magnetic properties than we had explored in the QM file, when we considered these two masses to be the same.

Anyway, so energies depend on Landau level, n, spin ms. But they’re degenerate w/r to ky. These degenerate wavefunctions are illustrated below (the strips). Each degenerate wavefunction corresponds to a particular ky = 2πny/Ly (ny being an integer here). Its center is at xcenter = sgn(e)kyℓB2, and ℓB is roughly the thickness of the wavefunction in the x-direction [but note that these wavefunctions overlap considerably and so ℓB is much larger than the apparent width of the strips, i.e., much greater than distance between successive xcenter’s – I just don’t know how to illustrate overlapping strips].

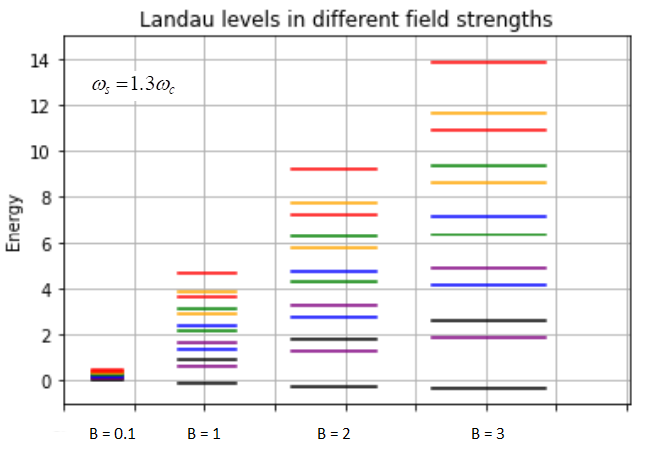


Degeneracy of the Landau level is just the number of degenerate wavefunctions/strips that fit into the sample [note due to the large wavefunction overlap, we cannot just say degeneracy is deg = Lx/ℓB] . It’s given by the condition:

:



where we’ve defined the flux quantum (it’s kind of disconcerting that deg can be 0. But I think the actual min is 1, or 2, or 3? Check out the QM folder/B field Symmetric gauge file – when you solve it in that gauge, the minimum degeneracy is not 0). Alternatively we can just think of the area occupied by a state to be Astate = 2πℓB2 or so, and therefore the total # of states we can stuff into the sample would be deg = A/Astate (well, this would be degeneracy *per* Landau level, note, i.e., per any *one* of those colored lines). And we’d note that the size (‘area’) of the state (2πℓB2 = 2π/|e|B) would decrease with B, which we might understand as being due to the magnetic field more tightly constraining the orbital radius of the state (having mind the classical orbital radius formula r = |e|v/meB). Made an energy level diagram, in no particular units, of the first 6 (n = 0, 1, 2, 3, 4, 5) Landau levels. Each is a different color: n = 0 is black, n = 1 is purple, n = 2 is blue, n = 3 is green, n = 4 is orange, n = 5 is red). Of course there are an infinite number of levels so should imagine them filling up the entire column. Each level comes in pair, due to spin splitting. The lower one is for spin down (therefore spin magnetic moment up), and the higher one is for spin up. I presumed the band mass to be slightly larger than the rest mass, so ωc < ωs. And can see this results in lowest energy level being less than 0, and so it drops with increasing magnetic field. Also drew the levels lengthening in proportion to the field, to illustrate how the degeneracy of the levels increases with B.



Note the levels will never cross: they start out coincident sort of (there is a continuum of levels in the B = 0 limit that stretch from E = 0 to E = ∞), at B = 0, but they all have different slopes (presumably) w/r to B, and so will all separate and continue drifting apart. For small B, the degeneracy is practically nill. And we’ll just have one electron in each level (or well, two if you include the spin splitting in the ‘level’). This is consistent with the free particle case, as the energy spectrum would be doubly degenerate in the spin directions for each distinct quantum state (px and py) as well. For large B, the degeneracy grows and we might have all electrons in just the bottom level eventually (if we’re in the ground state). What is the degeneracy, roughly, for a 2D substance, say? Well,



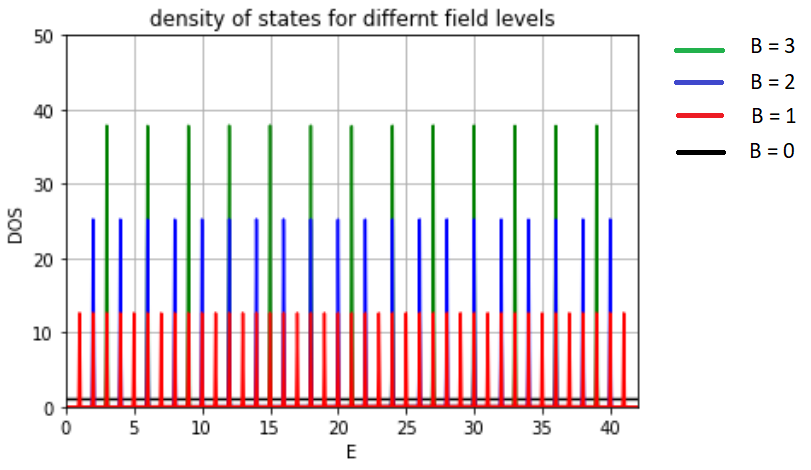
where N is the number of particles. So for fields ranging from B = 0T → 10T, we’ll go from N = roughly 0 (or 1) particles → 1000 particles per Landau level. And the number of Landau levels occupied will go from roughly N to N/1000. It’s definitely a lot either way. What is the spacing between levels? Well it’s Δε = ℏωB = ℏ|e|B/m = (7×10-4)B (eV).

**Density of states**

Now let’s look at the density of states (as have been doing, going to presume 2D substance so no degeneracy associated with kz). Also, leaving out spin for now: just looking at oribital d.o.f. So then the density of states would be defined as:



where A is area. Basically, it looks like this – just the energy diagram inverted.



At B = 0, it’s just a flat line, the typical 2D density of states. Let’s work this out. So in anyD, the density of states in the field free case, per unit volume (or area, length), was (see that Crystal Excitations file):



And in 2D this becomes:



sans some h-bar factors. But well, we’re not including spin here. So rather,



When turn on B. Then it transitions from smooth straight line, to bunch of discrete delta functions at each Landau level. The delta functions get further spaced as B increases, and also taller, as the degeneracy of each level increases. Each line represents an deg/A = (Φ/Φ0)/A = B/Φ0 fold degeneracy per unit area. One interesting observation we can make is the following: the average density of states is the same in the zero and non-zero field cases. Now in the field-full case, the spacing between energy levels is ωc



And the degeneracy of each level is:



So the kind of ‘average density of states’ in the field-full case is:

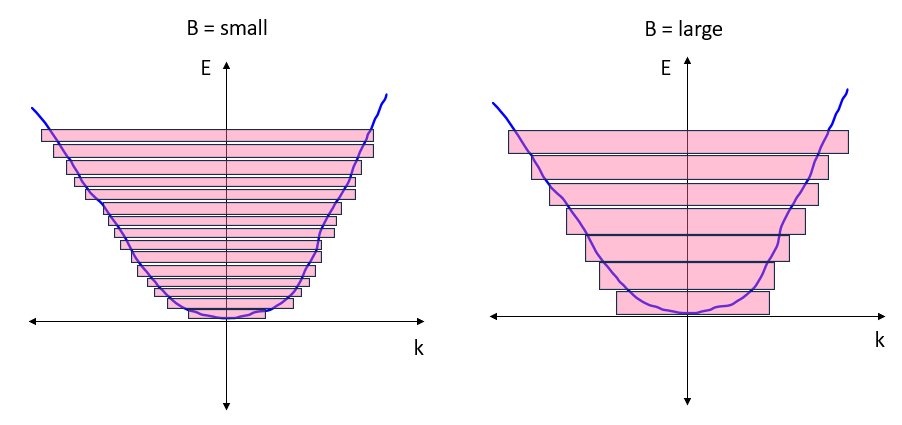


just as in the field-less case. Saw a nice picture of this somewhere, illustrating how the almost continuum of free states basically get squeezed into a the discrete Landau levels.

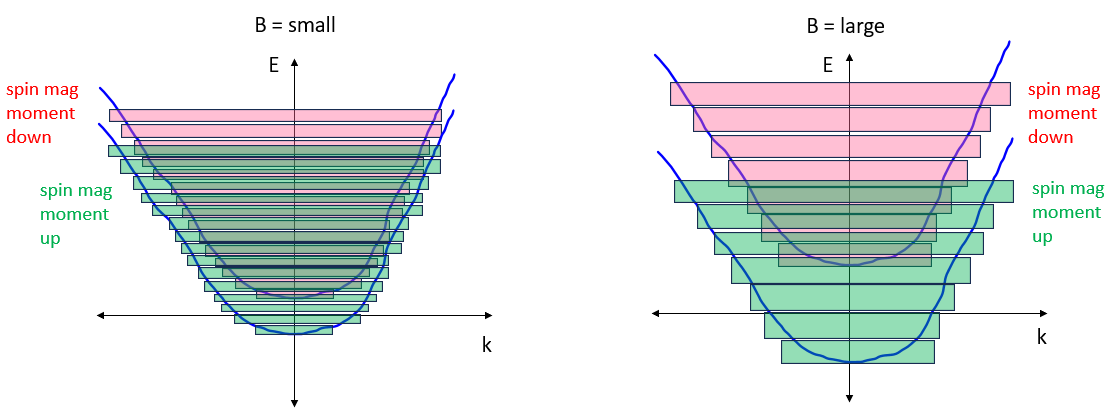
Diagram

Description automatically generated

So this picture, based on the density of states graph, is probably the best visualization. Just take the free particle spectrum, say a paraboloid if in 2D, and break it into cylindrical blocks of width ΔE = ωc – skinny blocks for small fields, and fat blocks for large fields. And the k-states encompassed by the blocks are the ones that get pulled into a degenerate Landau level. So the skinny blocks have low degeneracy, while the fat blocks have high degeneracy. And the energy of the Landau level would be the mean energy of the k-states encompassed within the block. I tried to illustrate it in 1D below. But note this diagram doesn’t account for the spin contribution to the energy.



If we account for spin, then the energy diagram would look something like this.



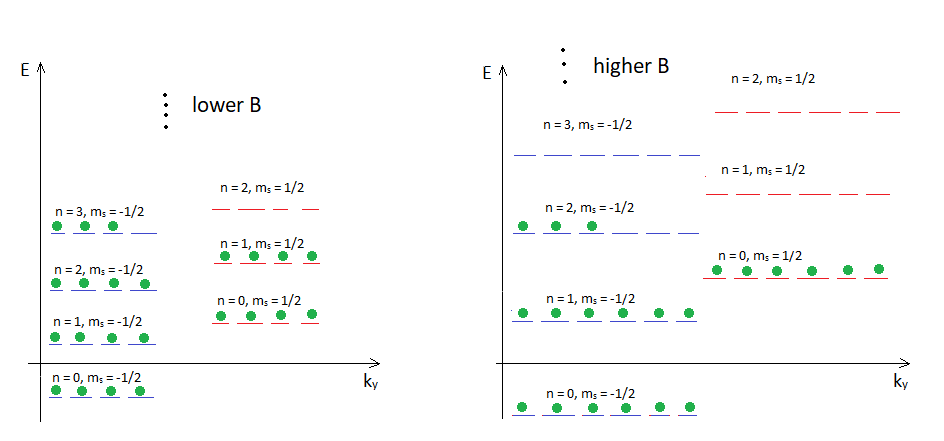
By spin up, I guess I mean ‘magnetic moment’ up, technically. Also, note each block has a *single* energy – the range of E’s encompassed by the thickness of the block is a measure of its degeneracy and the range of non-B states it has encompassed). We can define the density of particles (particles per area), n(μ), given a chemical potential, and fermion distribution function:



which we write as:



where ν is called the filling factor. ν basically gives the # of particles per 2πℓB2 of area regardless of energy. This number would roughly be the # of filled Landau levels. At T = 0, ν(μ) basically counts the number of Landau levels (including the spin-split ones) below μ. For instance, consider the following two diagrams showing the GS distribution of 23 particles in a hypothetical set of Landau levels, at different field strengths.



In the first, we’d have ν = 5.75, and in the second ν = 3.5. Looks like materials can be constructed for which ν = 1 in fact, i.e., all particles are in the lowest Landau level. Little surprised at that since our rough calculation above suggested this would require a field ~ 104T. But whatever.

**Some Expectations**

For instance, consider the velocity expectation of these states [note **v** = **p** – e**A**]:



Now let’s consider the expectation of the *physical* angular momentum in the z-direction. This is:



To get <x2> we’ll do something like (don’t need y or z part of wavefunction)



We can do that integral, as it’s just the expectation of PE = (1/2)mω2x2 divided by (1/2)mω2. And expectation of PE is half the expectation of the energy (since KE and PE are distributed equally in a HO). Of course the expectation of the energy is ωc(n+1/2). So,



Therefore we have:



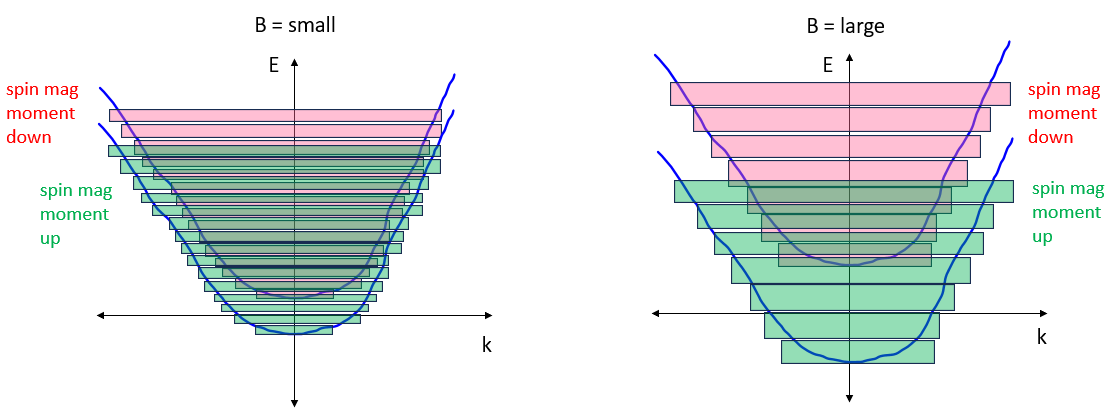
and so,



That’s a rather simple result. But it makes sense. Consider the magnetic moment: <Mz> = -∂<E>/∂B. So we have:



This is telling in that the orbital part of the magnetic moment is opposite the ambient field B, which indicates a diamagnetic response, regardless of the sign of the charge. The spin part of the response is paramagnetic though, since sgn(e)ms gives the direction of the magnetic moment, and so that part is always positive, i.e., in the direction of the field, as a paramagnetic response should be. This also is regardless the sign of the charge. We can use this to interpret our Landau levels a little better. Going back to that paraboloid energy diagram cylindrical block thing:



We can say that all the electrons in the cylindrical blocks are spinning diagmagnetically about the magnetic field lines (so if field is pointing up, then electrons are circulating CCW about the field lines, opposite to the way their spin magnetic moments are pointing). So the magnetic field takes the free spectrum electrons within the same energy block and alters their kx, ky values to go from criss-crossing the sample in rectalinear fashion to basically circulating around the magnetic field lines, thus giving the electrons a diamagnetic moment. And we’ll note that the electrons will circulate faster and faster as the energy level of the block increases. So the diamagnetic moment is increasing as we go up the energy diagram. This picture seems to indicate that the diamagnetic moment of a sample is incredibly large for small B’s, because M for each level just depends on n, and the degeneracy of each level would be super small for small B’s, so we’d have a lot of occupied n’s in a macroscopic sample.



On the other hand, since the degeneracy is, technically,



when B = 0, the degeneracy would ‘technically’ go to zero as well. So maybe this is just a delicate limit to handle.